Jackson Queuing Networks

AVACS S3
Phase 2

July 28, 2011

1 Description of the Benchmark

A Jackson Queuing Network (JQN) [2] is a system consisting of a number of n interconnected queuing stations. A JQN with two queues is depicted in Figure 1 Jobs arrive from the environment with a negative exponential inter-arrival time and are distributed to station i with probability $r_{0,i}$. Each station is connected to a single server which handles the jobs with a service time given by a negative exponential distribution with rate μ_i. Jobs processed by the station of queue i leave the system with probability $r_{i,0}$ but are put back into queue j with probability $r_{i,j}$. JQNs have an infinite state-space because the queues are unbounded. Initially all queues are empty. In this test case we consider JQN models with $N = 3, 4, 5$ queues. The arrival rate for N queues is λ, which is then distributed to station j (with service rate $\mu_j = j$) with probability $\frac{1}{\mu_j} \cdot \sum_{i=1}^{N} \mu_i$ The probability out of a service rate is then uniformly distributed. We compute the probability that, within $t = 10$ time units, a state is reached in which 4 or more jobs are in the first and 6 or more jobs are in the second queue.

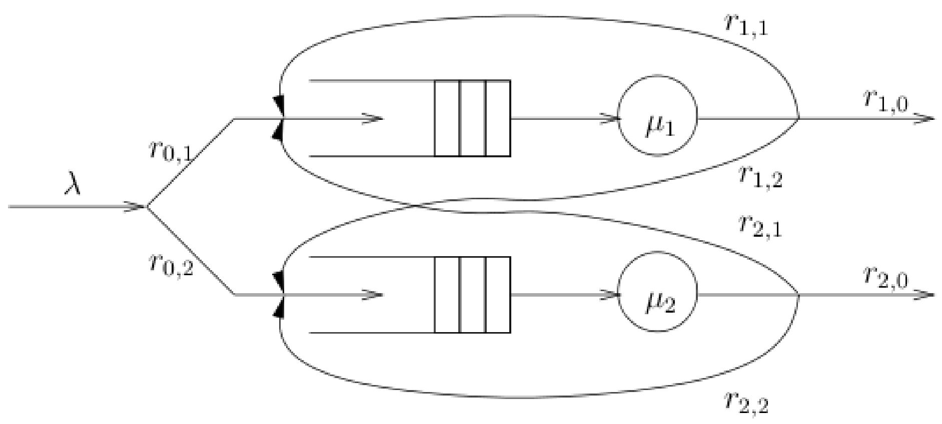

Figure 1: JQN with two queues

N/λ	Uniform dep	time	n	Layered dep	time	n	FSP dep	time	n
3 / 2	193	8/19	1236K	46	1/ 1	18K	37	4/ 0	10K
4 / 2	-	-	-	46	3/ 11	230K	35	43/ 4	82K
5 / 2	-	-	-	46	37/187	2349K	34	392/ 50	576K
3 / 3	203	10/23	1436K	60	1/ 1	40K	51	14/ 1	25K
4 / 3	-	-	-	60	7/ 27	635K	48	203/ 13	271K
5 / 3	-	-	-	-	-	-	46	2299/193	2349K
3 / 4	212	12/26	1633K	74	1/ 2	73K	64	35/ 1	48K
4 / 4	-	-	-	74	15/ 55	1426K	61	665/ 29	677K
3 / 5	223	13/31	1898K	88	2/ 3	121K	77	76/ 2	82K
4 / 5	-	-	-	88	30/101	2794K	74	1772/ 58	1426K

Table 1: Comparison of the model for Uniform, Layered-chain and FSP configuration

2 Results

We implemented the model in a *PRISM* [3]-like language and applied *INFAMY* [1] to it. Table 1 gives a comparison for three different configurations, Uniform, Layered-chain and FSP. The probability that, within $t = 10$ time units, a state is reached in which 4 or more jobs are in the first and 6 or more jobs are in the second queue was computed for different number of queues N and arrival rate λ. For the truncation, a precision of 10^{-6} was used. Results are given in Table 2. All results were obtained on a Linux machine with an AMD Athlon XP 2600+ processor at 2 GHz equipped with 2 GB of RAM.

Observe that for fixed arrival rate λ, the depth is insensitive to the number of queues in Layered and Uniform configurations. The service rate increases with N, however, transitions corresponding to the service rate only lead back to states which are already explored, thus not contributing to the forward exit rates. For FSP, the depth is smaller than the Layered-chain configuration. The number of states, and also the number of transitions, grow very fast with respect to the depth. Uniform chain cannot handle the cases $N = 4, 5$, as opposed to the other two configurations. While in the Layered-chain configuration the dominating part is the model checking time in the truncation, the dominating part of the FSP configuration is the state exploration part. For $N = 5$ and $\lambda = 3$, it is the only configuration that still works.

If the truncation grows very fast with respect to the depth, an approach that combines the two configurations might be the only option for large N and λ. Such an approach is left for future work.

N/λ	probability
3 / 2	1.98E-01
4 / 2	1.20E-01
5 / 2	8.94E-02
3 / 3	5.90E-01
4 / 3	4.42E-01
5 / 3	3.71E-01
3 / 4	8.38E-01
4 / 4	7.16E-01
3 / 5	9.43E-01
4 / 5	8.65E-01

Table 2: Probabilities of the property for different N and λ

References

[1] H. Hermanns, E. M. Hahn, B. Wachter, and L. Zhang. Time-Bounded Model Checking of Infinite-State Continuous-Time Markov Chains. *Fundamenta Informaticae*, 95:129–155, 2009.

[2] J.R. Jackson. Networks of Waiting Lines. *Operations Research*, 5:518–521, 1957.

[3] M. Kwiatkowska, G. Norman, and D. Parker. PRISM: Probabilistic Model Checking for Performance and Reliability Analysis. *ACM SIGMETRICS Performance Evaluation Review*, 36(4):40–45, 2009.